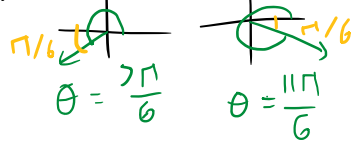


Warm-up!

A) Find solutions on the interval $0 \leq \theta < 2\pi$



$$\sin \theta = -\frac{1}{2}$$

B) Find ALL solutions

$$\theta = \frac{7\pi}{6} + 2\pi k \quad \theta = \frac{11\pi}{6} + 2\pi k, \text{ where } k \text{ is any integer}$$

$$\sin \theta = -\frac{1}{2}$$

C) Find ALL solutions

$$\sin(3\theta) = -\frac{1}{2}$$

$$\sin(X) = -\frac{1}{2}$$

$$X = \frac{7\pi}{6} + 2\pi k$$

$$3\theta = \frac{7\pi}{6} + 2\pi k$$

$$\theta = \frac{7\pi}{18} + \frac{2\pi k}{3}$$

$$\text{Let } X = 3\theta$$

$$X = \frac{11\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}$$

$$3\theta = \frac{11\pi}{6} + 2\pi k$$

$$\theta = \frac{11\pi}{18} + \frac{2\pi k}{3}, \quad k \in \mathbb{Z}$$

D) Find ALL solutions

$$\sin\left(\theta - \frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\sin(y) = -\frac{1}{2}$$

$$y = \frac{7\pi}{6} + 2\pi k$$

$$\theta - \frac{2\pi}{3} = \frac{7\pi}{6} + 2\pi k$$

$$\theta = \frac{11\pi}{6} + 2\pi k$$

$$\text{Let } y = \theta - \frac{2\pi}{3}$$

$$y = \frac{11\pi}{6} + 2\pi k$$

$$\theta - \frac{2\pi}{3} = \frac{11\pi}{6} + 2\pi k$$

$$\theta = \frac{15\pi}{6} + 2\pi k$$

Advanced Trig Day 3:
(3.10) Solving "multiple angle"
problems

Homework: Solving Multiple
Angle Equations + Review

Next Quiz: Day 6

- Solving Trig Equations
- Solving Inverse Trig Equations
- Trig Identities
- Quiz is on Day 1 - Day 4

$$\theta = \frac{11\pi}{6} + 2\pi k$$

$$\theta = \frac{5\pi}{2} + 2\pi k$$

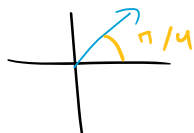
$$\theta = \frac{11\pi}{6} + 2\pi k$$

$$\theta = \frac{\pi}{2} + 2\pi k$$

Find ALL solutions

COS

$$\cos(y) = \frac{\sqrt{2}}{2}$$



$$y = \frac{\pi}{4} + 2\pi k$$

$$x + \frac{\pi}{2} = \frac{\pi}{4} + 2\pi k$$

$$x = -\frac{\pi}{4} + 2\pi k$$



$$y = \frac{7\pi}{4} + 2\pi k$$

$$x + \frac{\pi}{2} = \frac{7\pi}{4} + 2\pi k$$

$$x = \frac{5\pi}{4} + 2\pi k$$

$$\cos(x) \cos\left(\frac{\pi}{2}\right) - \sin(x) \sin\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}$$

$$\cos x \cdot 0 - \sin x \cdot 1 = \frac{\sqrt{2}}{2}$$

$$-\sin x = \frac{\sqrt{2}}{2}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$



$$x = \frac{5\pi}{4} + 2\pi k$$

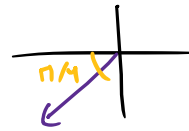
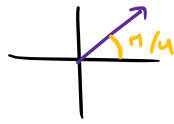


$$x = -\frac{\pi}{4} + 2\pi k$$

Find ALL solutions

$$\tan(2x) = 1$$

$$\tan(y) = 1$$



$$y = \frac{\pi}{4} + 2\pi k$$

$$y = \frac{5\pi}{4} + 2\pi k$$

$$2x = \frac{\pi}{4} + 2\pi k$$

$$x = \frac{\pi}{8} + \pi k$$

$$2x = \frac{5\pi}{4} + 2\pi k$$

$$x = \frac{5\pi}{8} + \pi k, k \in \mathbb{Z}$$

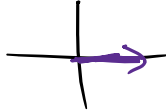
$$y = \frac{\pi}{4} + \pi k$$

$$2x = \frac{\pi}{4} + \pi k$$

$$x = \frac{\pi}{8} + \frac{\pi}{2} k$$

Then list only the solutions in the interval $0 \leq \theta < 2\pi$

$$3\theta - \pi = 2\pi n$$



$$y = 0 + 2\pi k$$

$$3\theta - \pi = 0 + 2\pi k$$

$$3\theta = \pi + 2\pi k$$

$$\Theta = \frac{\pi}{3} + \frac{2\pi k}{3}, \quad k \in \mathbb{Z}$$

Solutions in the interval $0 \leq \theta < 2\pi$:

Diagram illustrating the addition of angles for the 6th roots of unity:

$$\begin{array}{ccccccc}
 & & -2\pi/3 & & +2\pi/3 & & +2\pi/3 \\
 & \swarrow & \nwarrow & \swarrow & \nwarrow & \swarrow & \nwarrow \\
 -3\pi/3 & & -\pi/3 & & \pi/3 & & 3\pi/3 \\
 & & & & & & 5\pi/3 & & 7\pi/3
 \end{array}$$

The angles $\pi/3, \pi, 5\pi/3$ are highlighted in a box.

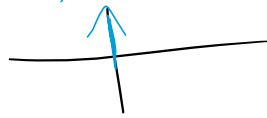
Find an expression for all x -values at which $y = \sec\left(\frac{1}{3}x\right)$ has vertical asymptotes.

$$y = \sec\left(\frac{1}{3}x\right)$$

$$y = \frac{1}{\cos\left(\frac{1}{3}x\right)}$$

$$\cos\left(\frac{1}{3}x\right) = 0$$

$$\cos(y) = 0$$



$$y = \frac{\pi}{2} + 2\pi k$$

$$\frac{1}{3}x = \frac{\pi}{2} + 2\pi k$$

$$x = \frac{3\pi}{2} + 6\pi k$$



$$y = \frac{3\pi}{2} + 2\pi k$$

$$\frac{1}{3}x = \frac{3\pi}{2} + 2\pi k$$

$$x = \frac{9\pi}{2} + 6\pi k$$

Find an expression for all x -values at which $y = \cot\left(x - \frac{\pi}{2}\right)$ has vertical asymptotes.

$$y = \cot\left(x - \frac{\pi}{2}\right)$$

$$y = \cot\left(x - \frac{\pi}{2}\right)$$

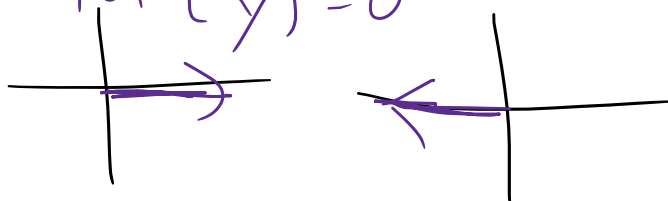
$$y = \frac{1}{\tan\left(x - \frac{\pi}{2}\right)}$$

$$y = \frac{\cos\left(x - \frac{\pi}{2}\right)}{\sin\left(x - \frac{\pi}{2}\right)}$$

$$\tan\left(x - \frac{\pi}{2}\right) = 0$$

$$\tan(y) = 0$$

$$\sin\left(x - \frac{\pi}{2}\right) = 0$$



$$y = 0 + 2\pi k$$

$$y = \pi + 2\pi k$$

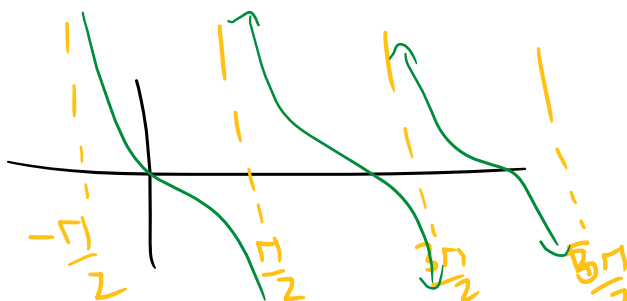
$$x - \frac{\pi}{2} = 0 + 2\pi k$$

$$x - \frac{\pi}{2} = \pi + 2\pi k$$

$$x = \frac{\pi}{2} + 2\pi k$$

$$x = \frac{3\pi}{2} + 2\pi k$$

$$x = \frac{\pi}{2} + \pi k$$



$$-\frac{\pi}{2} \quad | \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad \frac{5\pi}{2}$$

The function f is given by $f(\theta) = 2 \cos^2(2\theta) + 1$.

The function g is given by $g(\theta) = 3 \cos(2\theta)$.

Find an expression for all input values at which the graphs of f and g intersect.

Review of inverse trig equations

Solve each of the following:

$$\arctan(4x - 3) = -\frac{\pi}{4}$$

$$\sin^{-1}\left(\frac{x}{2}\right) = \arccos\left(\frac{1}{2}\right)$$

For each function, find an expression for the x -values of the vertical asymptotes.

$$f(x) = 3 \csc \left(x - \frac{\pi}{4} \right)$$

$$g(x) = \tan(4\pi x) - 2$$

Review of Advanced Trig Identities: Evaluate each of the following.

A) $\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{12}\right)\sin\left(\frac{\pi}{4}\right)$

B) $1 - 2\sin^2\left(\frac{7\pi}{8}\right)$

C) $30\sin\left(\frac{3\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)$

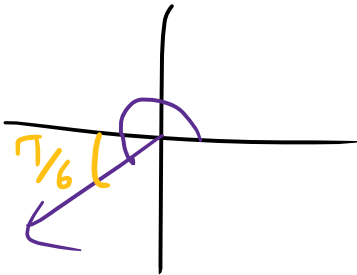
Find an expression for ALL solutions

$$6 \sin^2(3x) + 5 = 8$$

$$\sin(3\theta) = -\frac{1}{2}$$

$$\text{Let } 3\theta = X$$

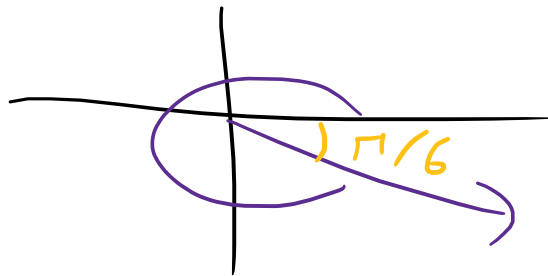
$$\sin(X) = -\frac{1}{2}$$



$$X = \frac{7\pi}{6} + 2\pi k$$

$$3\theta = \frac{7\pi}{6} + 2\pi k$$

$$\theta = \frac{7\pi}{18} + \frac{2\pi k}{3}$$



$$X = \frac{11\pi}{6} + 2\pi k$$

$$3\theta = \frac{11\pi}{6} + 2\pi k$$

$$\theta = \frac{11\pi}{18} + \frac{2\pi k}{3}$$